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特集 "Quantifier Elimination"

Parametric Robust Control by Quantifier Elimination

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Abstract

Recently for multi-objective robust control design problems the methods based on quantifier elimination (QE) have been proposed. We show efficient robust control design methods based on special quantifier elimination algorithms.

1 Introduction

It has been shown that many difficult problems in system and control theory can be solved by using quantifier elimination (QE) successfully[1)]. The first attempt by Anderson et al.[10] were made in 1970's, but at that time the algorithm of QE was very intricate and no appropriate software was available. In 1990's some improved algorithms have been developed by several authors and implemented on computers. By virtue of the considerable developments of both algorithms and software in QE methods, we can explore the application of the QE theory to control problems of great practical interest.

In this paper we focus on the application of QE to robust control design problems. Parametric robust control, in particular *a parameter space approach*, is known to be one of the effective methods for robust control synthesis and multi-objective design. The approach can be utilized to determine the set of certain parameters which satisfies the given specifications in a parameter space. However in general it is quite difficult to accomplish a parameter space approach for specifications of various kinds systematically. This deters many control engineers from applying parametric robust control methods to practically oriented field of industrial control. Therefore it is strongly desired to develop a method of accomplishing a parameter space approach systematically. QE enables us to do it.

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1)See [11, 1, 16, 23, 2, 4, 5, 6] for robust control design, [21, 22, 3] for verification of discrete time polynomial system and [19, 7] for verification of hybrid system.

In the last decade for multi-objective robust control synthesis a parameter space design accomplished by using QE has been proposed: The robust control problems are reduced to first-order formula descriptions, then they can be solved by applying a general QE algorithm based on cylindrical algebraic decomposition (CAD) [11, 16]. This covers a fairly wide range of robust control problems including many open problems. However, the naive reduction of control problems to QE problems, in general, is too complicated to achieve QE computation efficiently. This is a serious issue in view of efficiency because the worst-case complexity of general QE algorithm based on CAD algorithm has doubly exponential behavior. Aiming at practical QE-based approach we have proposed efficient methods which employ the scheme combining reduction of control problems to particular formulas and the usage of QE algorithms specialized to such particular formulas. Fortunately this still covers a quite wide range of robust control problems and has sufficient practicability (see [4, 5, 6]).

In this paper we show two successful special classes of such formulas in robust control: a definiteness of a univariate polynomial called a "*sign definite condition (SDC)*" and a set of linear constraints so called "*linear programming (LP)*". A special quantifier elimination based on the *Sturm-Habicht sequence* [12] is appropriate to solve SDC efficiently, while a special quantifier elimination based on *virtual substitution* [20] is suitable to solve parametric LP efficiently. We also present some examples from robust controller design problems. The experimental results in [4, 5] by both approaches using special QE algorithms via SDC and LP show that the scheme combining reduction of the problems to particular formulas and suitable special QE algorithms works in an efficient way for the practical control design problems, so called *fixed-order robust controller synthesis*, which is strongly needed in actual engineering fields and is a long-standing open problem in robust control. The framework via SDC and LP presented by the series of the works [4, 5, 6] provides a unifying platform for further research along this direction and also provides successful examples that shows QE can be an effective tool for actual industrial problems.

We have implemented the two special QE algorithms on Maple as SyNRAC [8]. QE computations in the examples of this paper finished within a few seconds on a Pentium III 1GHz CPU by using SyNRAC[2). Moreover, the implementation of MATLAB toolbox for parametric robust control based on the design methods shown in this paper by using SyNRAC as a kernel is planed [9]. See the paper by H.Yanami and H.Anai in this special issue on QE for the details of the implementation.

The rest of the paper is organized as follows: A parameter space approach is briefly explained in §2. Two special QE algorithms which are effective for parametric robust control are explained in §3. Robust controller synthesis based on special QE algorithms is presented in §4. Finally, in §5 we present our concluding remarks.

2) See [4, 5, 6, 8] for the detailed information of timing data.

2 Parametric robust control

In this section we show concrete description of target robust control problem "fixed-structure robust controller synthesis problems": Consider a feedback control system shown in Fig.1, where $\mathbf{p} = [p_1, p_2, \ldots, p_s]$ is the vector of uncertain real parameters in the plant G and $\mathbf{x} = [x_1, x_2, \ldots, x_t]$ is the vector of real parameters of the controller C. We consider the controllers of fixed-order. The performance of the control system can often be characterized

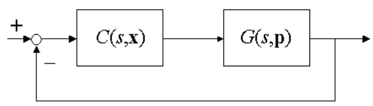

Figure 1: A standard feedback system

by a vector $\mathbf{a} = [a_1, \ldots, a_l]$ which are functions of the plant and controller parameters $\mathbf{p}$ and $\mathbf{x}$: $a_i = a_i(\mathbf{x}, \mathbf{p}), i = 1, \ldots, l$. Controller synthesis problems are to find $\mathbf{x}$ so that given specifications (e.g., $a_i(\mathbf{x}, \mathbf{p}) > \gamma_i,\ \gamma_i \in \mathbb{R}$) are satisfied. It is strongly desired that the fixed-order robust controller design problems is resolved in practical problems which operate under the constraint of fixed structure. However, it is a long-standing open problem in robust control and the lack of effective results on the problem has prevented modern design methods from being applicable to practical problems.

A parameter space approach based on QE has been proposed for such (multi-objective) robust controller synthesis [1, 11, 16, 2, 4]. The design scheme of this approach is as follows:

1. Determine the structure of the controller and select the design parameters, e.g., x_1 and x_2 in the PI-controller $x_1 + \frac{x_2}{s}$ are the parameters.

2. Reduce the specifications to the equivalent first-order formulas.

3. Compute the admissible regions of the design parameters for all specifications by applying QE to the obtained first-order formulas.

4. Superpose the admissible regions in the parameter space and take the parameters in the intersections.

We should avoid the *naive* reduction and simple application of general QE due to a drawback on computational complexity of CAD if possible. The scheme combining well the reduction to particular classes of formulas and usage of QE algorithms specialized to such classes is effective for the efficient computation. In fact this scheme is successfully applied to several open problems in robust control. Such examples are shown in §4.

3 Special QE methods

Here we show two QE algorithms specialized to particular types of input formulas that are successfully used to solve robust control design problems with practical efficiency.

3.1 A special QE using the Sturm-Habicht sequence

A special QE method based on the Sturm-Habicht sequence for the first-order formula $\forall x\ f(x) > 0$, where $f(x) \in \mathbb{R}[x]$ was proposed in [12]. The algorithm is desired to be modified for checking a *sign definite condition (SDC)*:

$$\forall x > 0,\ f(x) > 0, \tag{1}$$

since a quite wide range of the important problems in robust control can be reduced to SDC (see §4.1). We briefly sketch a special QE algorithm using the Sturm-Habicht sequence for the SDC (see [4] for details).

Definition 1
Let P, Q be polynomials in $\mathbb{R}[x]$; $P = \sum_{k=0}^{n} a_k x^k$, $Q = \sum_{k=0}^{m} b_k x^k$, where n, m are non-negative integers. For $i = 0, 1, \ldots, \ell = min(n, m)$ we define the subresultant associated to P, n, Q and m of index i by $Sres_i(P, n, Q, m)= \sum_{j=0}^{i} M_j^i(P, Q)x^j$, where $M_j^i(P, Q)$ is the determinant of the matrix composed by the columns $1, 2, \ldots, n+m-2i-1$ and $n+m-i-j$ in the matrix $s_i(P, n, Q, m)$:

$$s_i(P,n,Q,m) := \begin{array}{c} \overbrace{\hphantom{a_n \quad \cdots \quad a_0 \quad \cdots \quad a_0}}^{n+m-i} \\ \left(\begin{array}{ccccc} a_n & \cdots & a_0 & & \\ & \ddots & & \ddots & \\ & & a_n & \cdots & a_0 \\ b_m & \cdots & b_0 & & \\ & \ddots & & \ddots & \\ & & b_m & \cdots & b_0 \end{array}\right) \end{array} \begin{array}{l} \left.\vphantom{\begin{array}{c} a\\ a\\ a \end{array}}\right\} m-i \\ \left.\vphantom{\begin{array}{c} a\\ a\\ a \end{array}}\right\} n-i \end{array} .$$

Let $v = n + m - 1$ and $\delta_k = (-1)^{\frac{k(k+1)}{2}}$. The Sturm-Habicht sequence associated to P and Q is defined as the list of polynomials $\{SH_j(P, Q)\}_{j=0,\ldots,v+1}$ given by $SH_{v+1}(P, Q) = P$, $SH_v(P, Q) = P'Q$, $SH_j(P, Q) = \delta_{v-j} \cdot Sres_j(P, p, P'Q, v)$ for $j = 0, 1, \ldots, v-1$, where $P' = \frac{dP}{dx}$. In particular, $\{SH_j(P, 1)\}_{j=0,\ldots,v+1}$ is called the Sturm-Habicht sequence of P. We simply denote it by $\{SH_j(P)\}$.

The Sturm-Habicht sequence can be used for real root counting as is the Sturm sequence. Moreover it has better properties in terms of specialization of parameters and computational complexity (see [13, 14] for details):

Theorem 2 (González-Vega et al.[14])
Let $P(x) \in \mathbb{R}[x]$ and $\{g_0(x), \ldots, g_k(x)\}$ be a set of polynomials obtained from $\{SH_j(P(x))\}$ by deleting the identically zero polynomials. Let $\alpha, \beta \in \mathbb{R} \cup \{-\infty, +\infty\}$ s.t. $\alpha < \beta$. We define $W_{SH}(P;\alpha)$ as the number of sign variations on $\{g_0(\alpha), \ldots, g_k(\alpha)\}$. Then $W_{SH}(P;\alpha,\beta) \equiv W_{SH}(P;\alpha) - W_{SH}(P;\beta)$ gives the number of real roots of P in $[\alpha, \beta]$.

We denote the principal j-th Sturm-Habicht coefficient of $SH_j(f)$, i.e., the coefficient of degree j of $SH_j(f)$, by $st_j(f)$ and a constant term of $SH_j(f)$ by $ct_j(f)$ for all j. Then we have

$$\begin{aligned} W_{SH}(f;0,+\infty) &= W_{SH}(f;0) - W_{SH}(f;+\infty) \\ &= V(\{ct_n(f), \ldots, ct_0(f)\}) - V(\{st_n(f), \ldots, st_0(f)\}), \end{aligned} \tag{2}$$

where $V(\{a_i\})$ stands for the number of sign changes over the sequence $\{a_i\}$. The SDC (1) holds if and only if both W_{SH} $(f;0,\ +\infty) = 0$ and $st_n(f) > 0$ hold. Hence an equivalent condition to the SDC (1) can be obtained as follows: Consider all (at most) 3^{2n-1} possible sign combinations over the polynomials $ct_i(f)$,$st_i(f)$ since $ct_0(f) = st_0(f)$, $st_n(f) > 0, st_{n-1}(f) > 0$; Choose all sign conditions that satisfy $W_{SH}(f;0,+\infty) = 0$ by (2); Construct semi-algebraic sets generated by $ct_i(f)$, $st_i(f)$ for the selected sign conditions and combine them as a union. The obtained condition is of the form of a union of semi-algebraic sets [3)].

3.2 Linear QE by virtual substitution

We present another special QE algorithm, i.e., quantifier elimination for *linear formulas.* A linear formula is a formula whose atomic subformulas are all linear with respect to its quantified variables. We briefly show the QE algorithm for linear formulas by Weispfenning [24]. (See [20] for more efficient algorithms.)

Let $Q_1x_1 \cdots Q_nx_n\varphi$ be a linear formula, where $Q_i \in \{\forall, \exists\}$ and φ is a quantifier-free formula. By using the equivalence $\forall x\varphi(x) \Longleftrightarrow \neg(\exists x \neg\varphi(x))$, we can change the formula into an equivalent formula of the form $(\neg)\exists x_1 \cdots (\neg)\exists x_n(\neg)\varphi$. The negation '$\neg$' in $\neg\varphi$ with φ quantifier-free can be put into φ (use De Morgan's laws and rewrite the atomic subformulas), which is not essential part of quantifier elimination. We assume from now on that the input is an existential formula $\exists x_1 \cdots \exists x_n\varphi$ where φ is a quantifier-free formula. Now what we do is to eliminate the quantified variable $\exists x$ in $\exists x\varphi$ with φ quantifier-free.

Definition 3
Let φ be a quantifier-free formula, $x \in X$ a variable, and S a set of terms, where each term

[3)]The obtained result is expected to contain many empty sets. We can prune some impossible sign combinations beforehand (see [4]).

$t \in S$ does not contain x. Then S is called an elimination set for $\exists x \varphi$ if the equivalence

$$\exists x \varphi \Longleftrightarrow \bigvee_{t \in S} \varphi(x//t)$$

holds, where $\varphi(x//t)$ is the resulting formula obtained by substituting t for x [4).

Lemma 4 (Weispfenning [24])
Let φ be a linear quantifier-free formula, x a quantified variable in φ, and $\Psi = \{a_i x - b_i \ \rho_i \ 0 | i \in I, \rho_i \in \{=, \neq, \leq, <\}\}$ the set of atomic subformulas in φ. Then the following set S is an elimination set for $\exists x \varphi$:

$$S = \{b_i a_i^{-1}, b_i a_i^{-1} + 1, b_i a_i^{-1} - 1 | i \in I\} \cup \{1/2(b_i a_i^{-1} + b_j a_j^{-1}) | i, j \in I, i \neq j\} .$$

By using Lemma 4, we can eliminate all the quantifiers in a given formula (one by one from inside)[5). Using smaller elimination sets than in the above Lemma helps prevent the number of atomic subformulas from getting larger during the elimination process and hence improve the algorithm, see [20].

4 Control synthesis using special QE methods

4.1 Control system design via SDC

Many important design specifications in robust control such as

- H_∞ norm constraints (with/without a restriction on a frequency range) ,
- gain/phase margin constraints,
- pole assignment specifications,
- stability radius specification,

which are frequently used as indices for the robustness of feedback control systems, can be recast as SDCs: $\forall x > 0 \ f(x) > 0$ as shown in [15, 17, 18, 6, 4]. A special quantifier elimination using the Sturm-Habicht sequence has sufficient practicability for the SDCs derived from practical control problems, see [4, 6].

For example, an H_∞ norm constraint of a strictly proper transfer function $P(s) = n(s)/d(s)$ given by

$$\|P(s)\|_\infty := \sup_\omega |P(j\omega)| < 1$$

4) It is known that for any given linear formula $\exists x \varphi$ as above, there exists an elimination set for the formula.

5) The result obtained here may also contain many empty sets.

is equivalent to: $\forall\omega\ d(j\omega)d(-j\omega) > n(j\omega)n(-j\omega)$. Since we can find a function $f(\omega^2)$ which satisfies $f(\omega^2) = d(j\omega)d(-j\omega) - n(j\omega)n(-j\omega) > 0$, letting $x = \omega^2$ leads to SDC. Similarly, a finite frequency H_∞ norm defined by $\|P(s)\|_{[\omega_1,\omega_2]} := \sup_{\omega_1\le\omega\le\omega_2} |P(j\omega)| < 1$ can be recast as $f(x) \neq 0$ in $[\ -\omega_2^2, -\omega_1^2\ ]$, which is reduced to SDC for $f(z)$ by a bilinear transformation $z = -(x+\omega_2^2)/(x+\omega_1^2)$.

Example 1 (Hurwitz Stability with mixed sensitivity)
Consider the feedback system shown in Fig.1 with $G(s) = \frac{1}{s^2+s+1}$, $C(s,\mathbf{x}) = x_1 + \frac{x_2}{s}$. *The characteristic polynomial is given by* $g(s,\mathbf{x}) \equiv dn(G)dn(C) + nm(G)nm(C) = s^3 + s^2 + (x_1+1)s + x_2$, *where* $dn()$ *and* $nm()$ *mean a denominator and a numerator, respectively. Find the feasible set of parameter values of* $\mathbf{x}$ *in the PI controller* $C(s,\mathbf{x})$ *for the feedback system to achieve Hurwitz stability and the desired sensitivity.*

By Liénard-Chipart criterion, $g(s,\mathbf{x})$ *is Hurwitz iff*

$$\theta(\mathbf{x}) = (x_2 > 0\ \wedge\ x_1 - x_2 + 1 > 0) \tag{3}$$

holds. We also require the system to satisfy the finite frequency H_∞ *norms constraints of a sensitivity function* $S(s) = \frac{1}{1+G(s)C(s)}$ *and a complementary sensitivity function* $T(s) = \frac{G(s)C(s)}{1+G(s)C(s)}$ *simultaneously (This is called* mixed sensitivity problem*):*

$$||S(s)||_{[0,1]} \equiv \max_{0\le\omega\le1} ||S(\mathbf{i}\omega)|| < 0.1, \quad ||T(s)||_{[20,\infty]} \equiv \max_{20\le\omega\le\infty} ||T(\mathbf{i}\omega)|| < 0.05.$$

These specifications are also reduced to the following SDCs in z*, respectively:*

$$\forall z > 0\ \ (x_2^2-2x_2+x_1^2-99)z^3+(3x_2^2-4x_2+2x_1^2+2x_1-99)z^2+(3x_2^2-2x_2+x_1^2+2x_1-99)z+x_2^2 > 0,$$

$$\forall z > 0\ \ z^3+(-2x_1+1199)z^2+(-2x_2-399x_1^2-1598x_1+479201)z-399x_2^2-800x_2-159600x_1^2-319200x_1+63840400 > 0.$$

Performing QE to each SDC gives the following conditions in $\mathbf{x}$*:*

Sensitivity condition:

$$(P_3 \le 0 \wedge x_2 \neq 0) \vee (P_1 \ge 0 \wedge P_2 > 0) \vee (P_5 \ge 0 \wedge P_1 \ge 0 \wedge x_2 \neq 0) \tag{4}$$

where $P_1 = x_2^2 - 2x_2 + x_1^2 - 99$, $P_3 = x_1 + 11$, $P_5 = x_1 - 9$,

$$P_2 = 264627x_2^4 + 7128x_1x_2^3 - 349668x_2^3 - 3596x_1^3x_2^2 + 169274x_1^2x_2^2 + 462528x_1x_2^2 - 13152942x_2^2 + 2392x_1^4x_2 + 7952x_1^3x_2 - 426492x_1^2x_2 - 705672x_1x_2 + 19405980x_2 - 400x_1^6 - 1996x_1^5 + 105419x_1^4 + 352836x_1^3 - 9467766x_1^2 - 15524784x_1 + 288178803.$$

Complementary sensitivity condition:

$$P_6 \equiv 399x_2^2 + 800x_2 + 159600x_1^2 + 319200x_1 - 63840400 < 0. \tag{5}$$

The admissible region of $\mathbf{x}$ *which meets the all requirements is obtained by superposing (3),(4) and (5).*

4.2 Control system design via parametric LP

Recently it has reported [5] that other important problems in robust control, which are recast as parametric LP problems, can be resolved with sufficient efficiency for practical use by using a special QE method based on virtual substitution [24].

The method proposed in [5] makes it possible to estimate feasible relaxations of design specifications (for example, fixed-order robust pole assignment problem) exactly and systematically. This is important because when there is no feasible controller parameter value for a given specification, it is often required to relax the given specification within acceptable levels. We briefly show an example below to demonstrate how such problems are solved using the special QE for the class of linear formulas:

Example 2 (Possible relaxations of robust pole assignment specification [5]) *We consider a PI control system with $C(s) = x_1 + \frac{x_2}{s}$ for the plant $G(s) = \frac{1}{(d_2 s^2 + d_1 s + d_0)}$ where $-1 \leq d_0 \leq 1$, $1 \leq d_1 \leq 3/2$, $-1/2 \leq d_2 \leq 3/2$. The closed-loop characteristic polynomial is*

$$\delta(s) = d_2 s^3 + d_1 s^2 + (x_1 + c_0)s + x_2.$$

Then pole assignment problem is to locate the roots of $\delta(s)$ at(within) desired place(region). The target pole location is given as roots of a given target polynomial. Now we consider to estimate how much we can relax the given infeasible specification. The target (relaxed) characteristic polynomial is given by

$$\delta_T(s) = \delta_3^T s^3 + \delta_2^T s^2 + \delta_1^T s + \delta_0^T,$$

where $\delta_i^{T-} \leq \delta_i^T \leq \delta_i^{T+}$. Assume the endpoints are of the form: $\delta_i^{T-} = \sigma^i(\delta_i^0 - e_i\gamma)$, $\delta_i^{T+} = \sigma^i(\delta_i^0 + e_i\gamma)$ for all i where δ_i^0, e_i are given constants and σ and γ are parameters which stand for changes of the time-scale (or frequency range) and a magnitude of perturbations, respectively. Then we find the possible region of δ_i^{T-}, δ_i^{T+} (i.e., σ, γ) so that there exists a controller parameter $\mathbf{x}$ which satisfies that all roots of $\delta(s)$ are within the root space of $\delta_T(s)$. This is reduced to solve the following linear QE problem:

$$\exists x_1 \exists x_2 \exists d_0 \exists d_1 \exists d_2 \ \varphi(x_1, x_2, d_0, d_1, d_2, \sigma, \gamma),$$

$$\text{where } \varphi \equiv \begin{array}{l} ((\delta_3^{T-} \leq d_2 \leq \delta_3^{T+}) \wedge (\delta_2^{T-} \leq d_1 \leq \delta_2^{T+}) \wedge (\delta_1^{T-} \leq x_1 + d_0 \leq \delta_1^{T+}) \wedge \\ (\delta_0^{T-} \leq x_2 \leq \delta_0^{T+}) \wedge (-1 \leq d_0 \leq 1) \wedge (1 \leq d_1 \leq 3/2) \wedge (-1/2 \leq d_2 \leq 3/2)). \end{array}$$

Performing QE we obtain a quantifier free formula which shows the possible regions of σ, γ, that is, all possible relaxations in $\sigma - \gamma$ space. Then we can easily evaluate the minimum or other appropriate relaxations (see [5] for a concrete example).

5 Conclusion

In this paper we introduced efficient robust control design methods by a parameter space approach based on special QE algorithms. Actually, many robust control design problems are solved efficiently by the QE-based approach as shown in [4, 5, 6, 8]. Combining reduction of control problems to particular formulas and usage of QE algorithms specialized to such particular formulas works in an efficient way for the practical control design problems. This provides a nice example of practical application of QE.

One of the important future works is to provide a software tool for control designers. The implementation of MATLAB toolbox for parametric robust control based on SyNRAC is planed [9].

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