# Counting Young Group Double Cosets with Computer Algebra

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**Abstract**

Until the advent of computer algebra, the theory of double cosets has been restricted to a few elegant but computationally impossible theorems. Impossible in the sense that in principle the calculation can be done but it will take ten thousand years. Today, using Computer Algebra much can be calculated quickly. Using Macsyma and Maple in the special case of Young Group double cosets, we will see just how valuable Computer Algebra can be.

We will focus on the use of CA in three areas of interest: the initial calculations, the generation of novel double coset symbols, and the generalization the method of successive subtractions.

Without CA the new insights into the theory of double cosets would never have been discovered. There was no particular reason to believe that the long arduous calculations required to compute a each double coset number would yield any fruit. Thanks to the power of MACSYMA these calculations were simple to conduct. The result was a breakthrough that immediately revealed a multitude of patterns.

In follow-up research a novel system of double coset symbols was developed. The novel system uses a special canonical form that can be expressed in Maple as a subroutine. Using this subroutine and other Maple subroutines, lists of these symbols can be quickly generated.

In the initial calculations, successive subtractions of the data eventually yielded a constant: thus uncovering a data fitting polynomial (in this case a quartic). The initial calculations used successive subtractions to point at the pattern, but the pattern itself is not a proof. However, the method of successive subtractions also points to the next step. Using the lists of DC-symbols and Maple once again the method of successive subtractions is generalized by successive subtractions of sets. The patterns revealed resulted in a series of proofs.

Time permitting; sketches of the proofs will be given with special emphasis on the roles of Macsyma and Maple. It is easy to see that in all these cases, CA was a critical aid in speeding the requisite insight for the next leap in theory.