

Fist Order Algebraic Differential Equations – A Computer Algebraic Approach

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Abstract

In this talk, we present our computer algebraic approach to first order algebraic differential equations. We apply the algebro geometric approach to the study of first order algebraic differential equations and computer algebraic approach is given. The algebro geometric approach is used to obtain bound of the degree of rational solutions of a first order algebraic differential equation with algebraic genus greater than one and the number of rational solutions of a first order algebraic differential equation. Matsuda's and Eremenko's algorithms are explicitly given by computer algebra. The algebraic general solutions of first order algebraic differential equations were studied by using of the birational transformations of algebraic curves, and an algorithm was presented to get an algebraic general solution of first order algebraic differential equations without movable critical point if the algebraic general solution exists. We also present a polynomial algorithm for the uniform solutions of first order algebraic differential equations with constant coefficients. All of the algorithms are implemented by Maple.

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Acknowledgment

The author is partially supported by the NKBRSF of China (No. 2004CB318000), the NNSF (No. 10301032) and by a CNRS—K. C. WONG fellowship during his visit to the Laboratoire P. Painlevé, Université de Lille 1, France.