A Unified Formula for Arbitrary Order Symbolic Derivatives and Integrals of The Power-Exponential Class

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Abstract

We give a complete solution to the problem of symbolic differentiation and integration of arbitrary (integer, fractional, or real) order of the *power-exponential class*

$$\left\{ f(x) : f(x) = \sum_{j=1}^{\ell} p_j(x^{\alpha_j}) e^{\beta_j x^{\gamma_j}} \,, \alpha_j \in \mathbb{C}, \beta_j \in \mathbb{C} \backslash \{0\}, \gamma_j \in \mathbb{R} \backslash \{0\} \right\} ,$$

through a unified formula in terms of the H-function which can, in many cases, be simplified to less general functions. We begin our talk by discussing a less general class of functions given by

$$\left\{ f(x) : f(x) = \sum_{j=1}^{\ell} p_j(x) e^{\beta_j x} \,, \beta_j \in \mathbb{C} \right\} ,$$

which is a subclass of the *power-exponential class*. It has the property that its nth derivative and integral formulas of integer order belongs to the same class.

In Maple, the formulas correspond to invoking the commands $\mathrm{diff}(f(x), x\$q)$ for differentiation and $\mathrm{int}(f(x), x\$q)$ for integration, where q is an integer, a fraction, a real, or a symbol. They enhance the ability of computer algebra systems for computing derivatives and integrals of very large arbitrary orders at a point x.

The arbitrary order of differentiation is found according to the Riemann-Liouville definition, whereas the generalized Cauchy n-fold integral is adopted for arbitrary order of integration.

One of the key points in this work is that the approach does not depend on integration techniques.