Space-Efficient Evaluation of Hypergeometric Series

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Abstract

Hypergeometric series are used to approximate many important constants, such as e and Apéry's constant $\zeta(3)$. The evaluation of such series to high precision has traditionally been done by binary splitting followed by integer division. For evaluating such a series to N digits of accuracy, the numerator and the denominator computed by binary splitting have size $O(N \log N)$ even though the reduced fraction may only have size $O(N)$.

In this talk, we show how standard computer algebra techniques including modular computation and rational reconstruction can be applied. The space complexity of our algorithm is the same as a bound on the size of the reduced fraction of the series approximation. We also show that the series approximation for $\zeta(3)$ is indeed a reduced fraction of size $O(N)$. The analysis can be related to our previous algorithm using partially factored integers (ISSAC 2000), which in turn allows our previous algorithm to be improved as well (Hanrot, Thomé, Zimmermann). This technique is also applicable to a large class of hypergeometric series.

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