Perturbation Expansion Techniques to Solution of Fluid Flow Problem Using Symbolic Computation

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Abstract

The linearised equations and boundary conditions governing the onset of Bénard-Marangoni convection in a fluid layer subject to uniform internal heat generation are given by the boundary value problem,

$$(D^2-a^2)\left(D^2-a^2-\frac{s}{P_r}\right)W(z)-a^2R\Theta(z) = 0, \quad (1)$$

$$(D^2-a^2-s)\Theta(z)+[1-Q(1-2z)]W(z) = 0, \quad (2)$$

subject to

$$sf-W(1) = 0, \quad (3)$$

$$C_r\left(D^2-3a^2-\frac{s}{P_r}\right)DW(1)-a^2(a^2+B_o)f = 0, \quad (4)$$

$$(D^2+a^2)W(1)+a^2\Gamma R[\Theta(1)-(1+Q)f] = 0, \quad (5)$$

$$D\Theta(1)+B_i[\Theta(1)-(1+Q)f] = 0, \quad (6)$$

and

$$W(0)=0, \quad DW(0)=0, \quad \Theta(0)=0, \quad (7)$$

where $D=\mathrm{d}/\mathrm{d}z$. Since the exact solution to the full problem (1)–(7) are difficult to obtain, we seek solutions for W, R, $\omega=\mathrm{Im(s)}$, Θ and f by perturbation expansions in the forms $W(Z)=\sum_{i=0}^{\infty}\epsilon^i W_i(Z)$, $R=\epsilon^{-4}\sum_{i=0}^{\infty}\epsilon^i R_i$, $\omega=\epsilon^{-3}\sum_{i=0}^{\infty}\epsilon^i\omega_i$, $\Theta(Z)=\sum_{i=0}^{\infty}\epsilon^i\Theta_i(Z)$ and $f=\epsilon^2\sum_{i=0}^{\infty}\epsilon^i f_i$, where we have introduced the new small parameter $\epsilon=a^{-1/2}$ and $1-z=Z/a$, via a computer algebra system. In this work, we present an example of an application in which large expressions in the symbolic computations can arise.