

# Perturbation Expansion Techniques to Solution of Fluid Flow Problem Using Symbolic Computation

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## Abstract

The linearised equations and boundary conditions governing the onset of Bénard-Marangoni convection in a fluid layer subject to uniform internal heat generation are given by the boundary value problem,

$$(D^2 - a^2) \left( D^2 - a^2 - \frac{s}{P_r} \right) W(z) - a^2 R \Theta(z) = 0, \quad (1)$$

$$(D^2 - a^2 - s) \Theta(z) + [1 - Q(1 - 2z)] W(z) = 0, \quad (2)$$

subject to

$$sf - W(1) = 0, \quad (3)$$

$$C_r \left( D^2 - 3a^2 - \frac{s}{P_r} \right) DW(1) - a^2(a^2 + B_o)f = 0, \quad (4)$$

$$(D^2 + a^2)W(1) + a^2 \Gamma R[\Theta(1) - (1 + Q)f] = 0, \quad (5)$$

$$D\Theta(1) + B_i[\Theta(1) - (1 + Q)f] = 0, \quad (6)$$

and

$$W(0) = 0, \quad DW(0) = 0, \quad \Theta(0) = 0, \quad (7)$$

where  $D = d/dz$ . Since the exact solution to the full problem (1)–(7) are difficult to obtain, we seek solutions for  $W$ ,  $R$ ,  $\omega = \text{Im}(s)$ ,  $\Theta$  and  $f$  by perturbation expansions in the forms  $W(Z) = \sum_{i=0}^{\infty} \epsilon^i W_i(Z)$ ,  $R = \epsilon^{-4} \sum_{i=0}^{\infty} \epsilon^i R_i$ ,  $\omega = \epsilon^{-3} \sum_{i=0}^{\infty} \epsilon^i \omega_i$ ,  $\Theta(Z) = \sum_{i=0}^{\infty} \epsilon^i \Theta_i(Z)$  and  $f = \epsilon^2 \sum_{i=0}^{\infty} \epsilon^i f_i$ , where we have introduced the new small parameter  $\epsilon = a^{-1/2}$  and  $1 - z = Z/a$ , via a computer algebra system. In this work, we present an example of an application in which large expressions in the symbolic computations can arise.